

# Geometry: 11.1-11.4 Notes

NAME \_\_\_\_\_

## 11.1 Circumference and Arc Length

Date: \_\_\_\_\_

### Define Vocabulary:

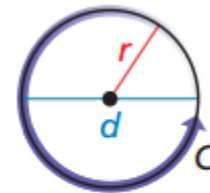
circumference –

arc length –

radian –

### Circumference of a Circle

The circumference  $C$  of a circle is  $C = \pi d$  or  $C = 2\pi r$ , where  $d$  is the diameter of the circle and  $r$  is the radius of the circle.



$$C = \pi d = 2\pi r$$

### Examples: Using the Formula for Circumference

#### WE DO

Find each indicated measure.

a. circumference of a circle with a radius of 11 inches

b. radius of a circle with a circumference of 4 millimeters

#### YOU DO

1. Find the circumference of a circle with a diameter of 5 inches.

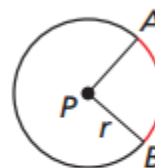
2. Find the diameter of a circle with a circumference of 17 feet.

### Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to  $360^\circ$ .

$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

$$\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

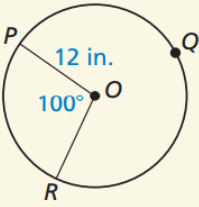


**Examples: Using Arc Lengths to Find Measures**

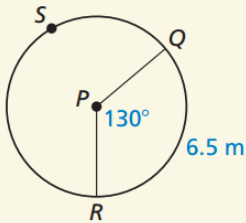
**WE DO**

Find each indicated measure.

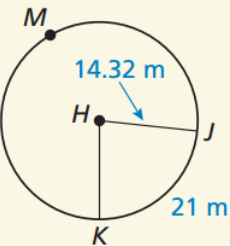
a. arc length of  $\widehat{PR}$



b. circumference of  $\odot P$



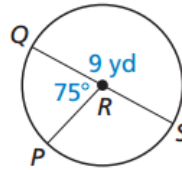
c.  $m\widehat{JK}$



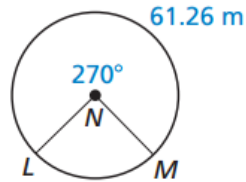
**YOU DO**

Find the indicated measure.

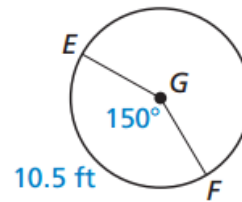
arc length of  $\widehat{PQ}$



circumference of  $\odot N$



radius of  $\odot G$



### Examples: Using Circumference to Find Distance Traveled

#### WE DO

The radius of a wheel on a toy truck is 4 inches.  
To the nearest foot, how far does the wheel travel  
when it makes 7 revolutions?

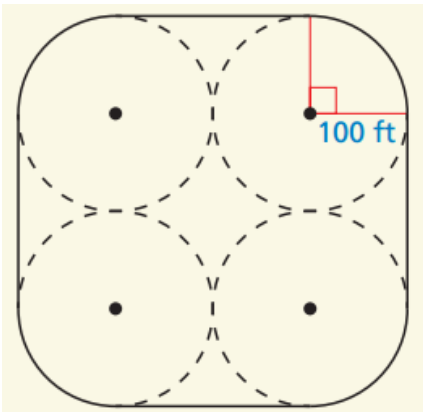
#### YOU DO

A car tire has a diameter of 28 inches. How  
many revolutions does the tire make while  
traveling 500 feet?

### Examples: Using Arc Length to Find Distances

#### WE DO

A path is built around four congruent circular  
fields. The radius of each field is 100 feet. How  
long is the path? Round to the nearest hundred feet.



## Converting between Degrees and Radians

### Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}.$$

### Radians to degrees

Multiply radian measure by

$$\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}.$$

### Examples: Converting between Degree and Radian Measure

#### WE DO

a. Convert  $30^\circ$  to radians.

b. Convert  $\frac{3\pi}{8}$  radians to degrees.

#### YOU DO

1. Convert  $15^\circ$  to radians.

2. Convert  $\frac{4\pi}{3}$  radians to degrees.

Assignment	
------------	--

**Define Vocabulary:**

population density –

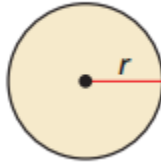
sector of a circle –

**Area of a Circle**

The area of a circle is

$$A = \pi r^2$$

where  $r$  is the radius of the circle.

**WE DO**

Find each indicated measure.

- area of a circle with a radius of 8.5 inches
- diameter of a circle with an area of 153.94 square feet

**YOU DO**

Find the indicated measure.

- Find the area of a circle with a radius of 4.5 meters.
- Find the radius of a circle with an area of 176.7 square feet.

**Using the Formula for Population Density**

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

## Examples: Using the Formula for Population Density

### WE DO

a. About 124,000 people live in a 2-mile radius of a city's post office. Find the population density in people per square mile.

b. A region with a 10-mile radius has a population density of about 869 people per square mile. Find the number of people who live in the region.

c. About 150,000 people live in a circular region with a population density of about 1578 people per square mile. Find the radius of the region.

### YOU DO

1. About 58,000 people live in a region with a 2-mile radius. Find the population density in people per square mile.

2. A region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.

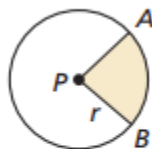
3. About 1.75 million people live in a circular region with a population density of about 5050 people per square mile. Find the radius of the region.

### Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle ( $\pi r^2$ ) is equal to the ratio of the measure of the intercepted arc to  $360^\circ$ .

$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

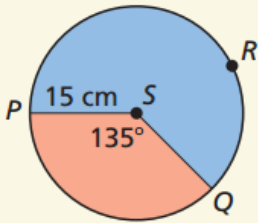
$$\text{Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$



**Examples: Finding Areas of Sectors**

**WE DO**

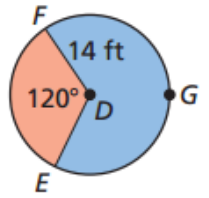
Find the areas of the sectors formed by  $\angle PSQ$ .



**YOU DO**

Find the indicated measure.

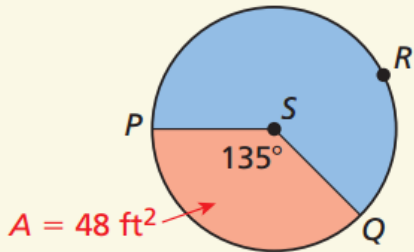
5. area of red sector
6. area of blue sector



**Examples: Using the Area of a Sector**

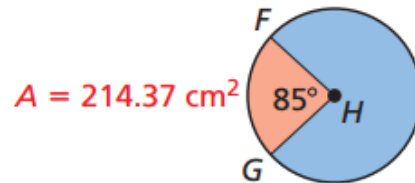
**WE DO**

Find the area of  $\odot S$ .



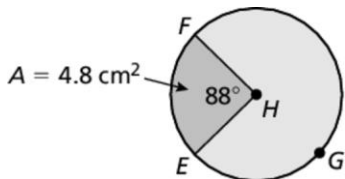
**YOU DO**

Find the area of  $\odot H$ .



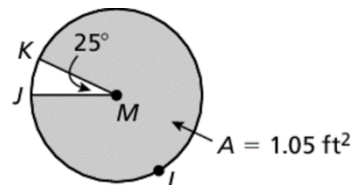
**WE DO**

Find the radius of circle  $H$



**YOU DO**

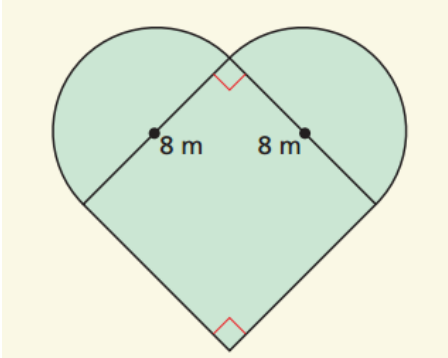
Find the radius of circle  $M$



## Examples: Finding the Area of a Region

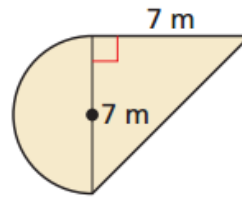
### WE DO

A farmer has a field with the shape shown. Find the area of the shaded region to the nearest square meter.



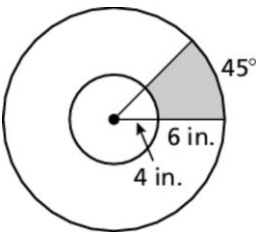
### YOU DO

Find the area of the figure.



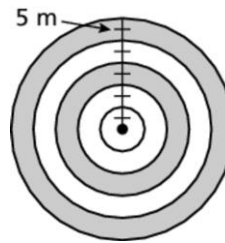
### WE DO

Find the area of the shaded region



### YOU DO

Find the area of the shaded region



Assignment	
------------	--



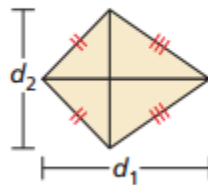
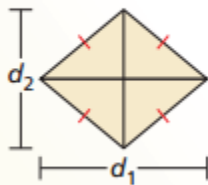
**Define Vocabulary:**

center of a regular polygon –

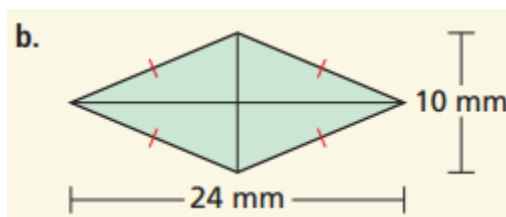
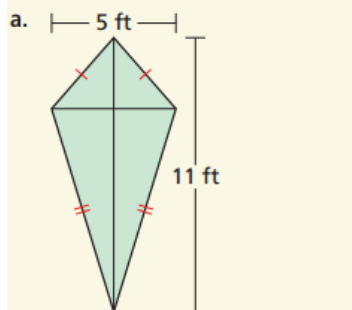
radius of a regular polygon –

apothem of a regular polygon –

central angle of a regular polygon –

**Area of a Rhombus or Kite**The area of a rhombus or kite with diagonals  $d_1$  and  $d_2$  is  $\frac{1}{2}d_1d_2$ .**Examples: Finding the Area of a Rhombus or Kite****WE DO**

Find the area of each rhombus or kite.

**YOU DO**

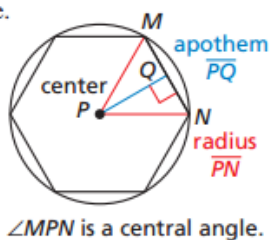
1. Find the area of a rhombus with diagonals  $d_1 = 4$  feet and  $d_2 = 5$  feet.
2. Find the area of a kite with diagonals  $d_1 = 12$  inches and  $d_2 = 9$  inches.

## Finding Angle Measures in Regular Polygons

The diagram shows a regular polygon inscribed in a circle.

The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word "apothem" refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

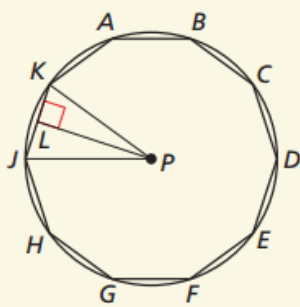


A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide  $360^\circ$  by the number of sides.

### Examples: Finding Angle Measures in a Regular Polygon

#### WE DO

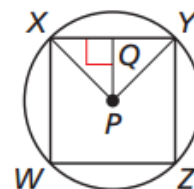
In the diagram, polygon  $ABCDEFGHIJK$  is a regular decagon inscribed in  $\odot P$ . Find each angle measure.



- $m\angle KPJ$
- $m\angle LPK$
- $m\angle LJP$

#### YOU DO

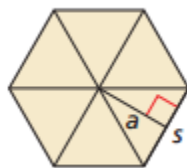
- Identify the center, a radius, an apothem, and a central angle of the polygon.
- Find  $m\angle XPY$ ,  $m\angle XPQ$ , and  $m\angle PXQ$ .



### Area of a Regular Polygon

The area of a regular  $n$ -gon with side length  $s$  is one-half the product of the apothem  $a$  and the perimeter  $P$ .

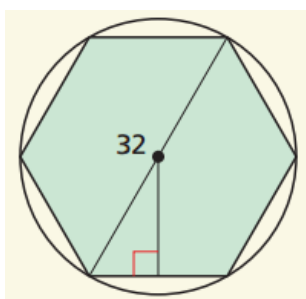
$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$



### Examples: Finding the Area of a Regular Polygon

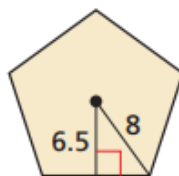
#### WE DO

A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.



#### YOU DO

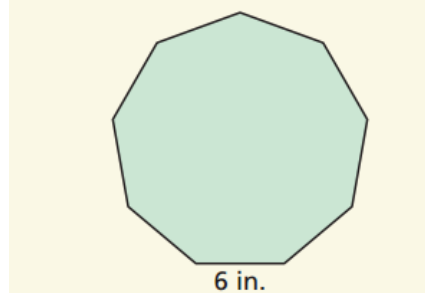
Find the area of the regular polygon.



### Examples: Finding the Area of a Regular Polygon

#### WE DO

A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?



Assignment	
------------	--

**Define Vocabulary:**

polyhedron – A solid that is bounded by polygons

face – A flat surface of a polyhedron

edge – A line segment formed by the intersection of two faces of a polyhedron

vertex (of a polyhedron) – A point of a polyhedron where three or more edges meet

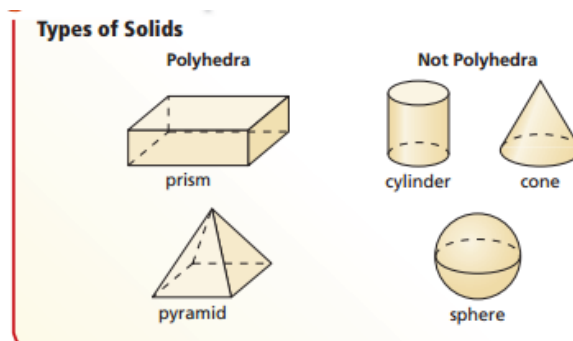
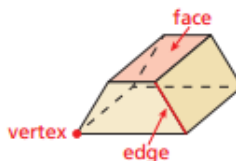
cross section – The intersection of a plane and a solid

solid of revolution – A three dimensional figure that is formed by rotating a two dimensional shape around an axis

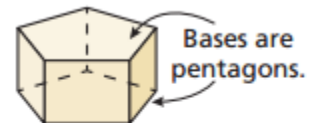
axis of revolution – The line around which a two dimensional shape is rotated to form a three-dimensional figure

**Classifying Solids**

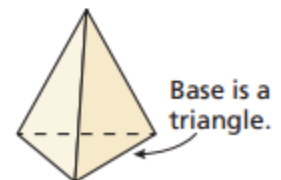
A three-dimensional figure, or solid, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



Pentagonal prism



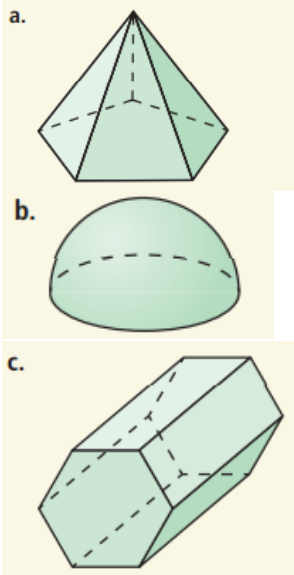
Triangular pyramid



## Examples: Classifying Solids

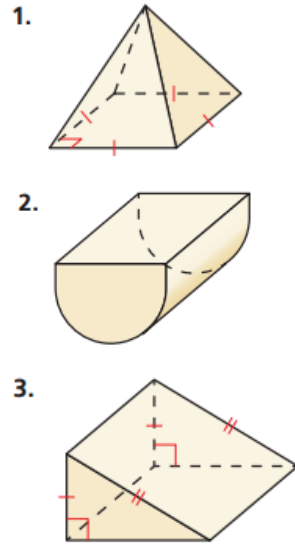
### WE DO

Tell whether each solid is a polyhedron. If it is, name the polyhedron.



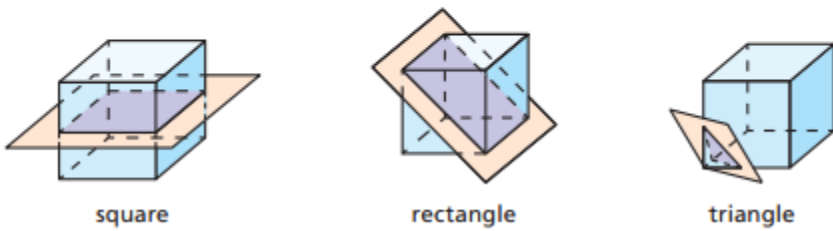
### YOU DO

Tell whether each solid is a polyhedron. If it is, name the polyhedron



## Describing Cross Sections

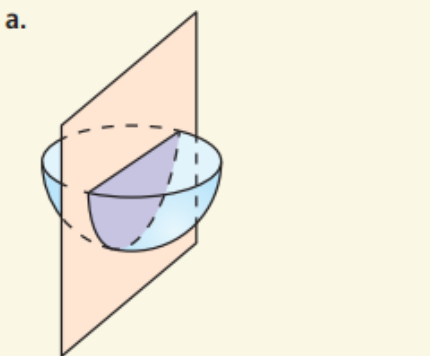
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



## Examples: Describing Cross Sections

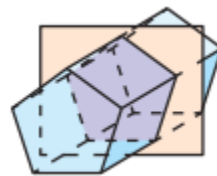
### WE DO

Describe the shape formed by the intersection of the plane and the solid.



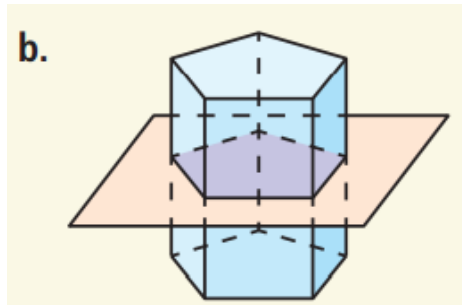
### YOU DO

Describe the shape formed by the intersection of the plane and the solid.



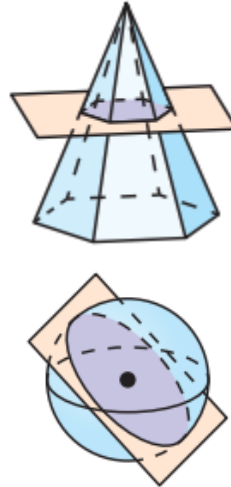
**WE DO**

Describe the shape formed by the intersection of the plane and the solid.



**YOU DO**

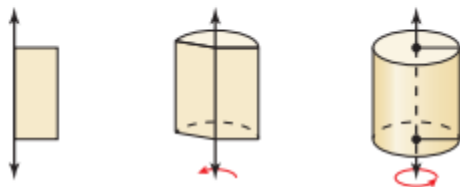
Describe the shape formed by the intersection of the plane and the solid.



**Sketching and Describing Solids of Revolution**

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

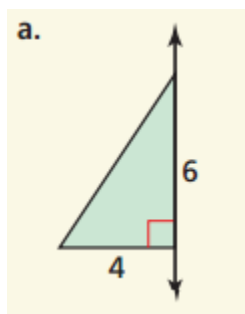
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.



**Examples: Describing Cross Sections**

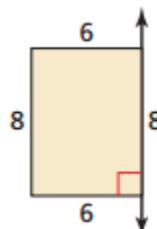
**WE DO**

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.



**YOU DO**

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.



Assignment	
------------	--

